Comment on the statistical analysis in "A new experimental limit for the stability of the electron" by H.V. Klapdor-Kleingrothaus, I.V. Krivosheina and I.V. Titkova

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Abstract

We have revealed evident errors in the statistical analysis, performed by Klapdor-Kleingrothaus et al in a recently published paper [1] to establish a limit on the stability of electron with respect to the decay into $\nu + \gamma$. The performed reestimation of the sensitivity of the experimental setups to the 256 keV gamma emitted in the hypothetical electron decay, has shown that the limits on the electron stability and charge nonconservation parameter $\epsilon^2_{e\nu\gamma}$ presented in [1], have been overestimated by at least a factor of 5.

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Incongruity of the analysis is evident already from the comparison of the sensitivity estimate with the "1σ" method and the results of the maximum likelihood and χ^2- analysis, the latter two give the result by a factor of 3-5 better. The most evident manifestation of errors in the analysis can bee seen in Table 7 of Ref.[1] (called hereafter the KKKT article):

1. The best fit of the ANG2 data set contains 89.444 ± 63.058 events[1] in the peak corresponding to ~256 keV ($m_e/2$) γ from hypothetical electron decay $e \rightarrow \nu + \gamma$ for the ME case[2]. As follows from the text of the article the error in this number is cited at 68% c.l. (it is called an indication of a signal on a 1.4 σ c.l.). The corresponding upper limit on the number of events in the peak, λ, already at 50% c.l., should be higher than the central value of 89 in most practical cases of any almost-symmetrical χ^2- profile shapes. Instead of the above it is claimed to be only 38 events at 68% c.l.

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[1] We are reproducing the number of significant digits, following the original text of the KKKT article.

[2] See the original text of the KKKT article [1] to explain the abbreviations, we are also citing a number of values from the KKKT article without going too much into details.

2. The upper limit on the number of events in the 256 keV peak obtained for ANG4 set, is 4.789 events at 68% c.l. This value is lower than 1σ statistical error on the content of a single bin in Fig. 8d, while the FWHM peak width is about 20 bins and the mean bin content is > 50 events.

These evident errors, together with the fact that an "indication of a signal on 1.4 σ" for 1.94×10^{26} yr is excluded both by the results of the Borexino [2] and DAMA [3] collaborations with more than 90% probability, convinced us to have a closer look at the results of KKKT. The results from the KKKT article are summarized in Table 1 (the data are taken from Tables 5 and 7 of the KKKT article; for the sake of simplicity we are citing only the ME case at 68% c.l.). These are the results of the analysis of experimental data with the 1σ method ("1σ" column) and with the standard least square procedure ("χ^2" column). One can see incompatibility of the values in N_{peak} and λ subcolumns, the most evident discrepancies are described above.

In order to check the achievable sensitivity when looking for the gaussian shape on the linear background, we have applied a toy model consisting of a gaussian peak superimposed on the linear background. For the toy model we used the background level for the corresponding data set from the KKKT article, the region of analysis was set to 100 keV with the bin width of 0.36 keV, and the 1σ width of the gaussian peak corresponded to the Doppler-broadened line of 3.25 keV. The number of the events that can be eliminated at 1σ level was defined for a large number of samples by using the MC method. For each sample a set of randomly distributed events was simulated with the fixed mean number of events in the gaussian peak, then we fitted it with linear+Gauss analytical shape. The sensitivity at 68% confidence level in this approach corresponds to the mean number of events in the peak for which the χ^2 value increases by $\Delta\chi^2 = 1$. The results are presented in "MC" column of Table 2.

Comparing the λ values in MC column to the KKKT 1σ estimate one can see that the toy model gives the same level of sensitivity, confirming the KKKT estimation with the 1σ method. Nevertheless, the values of λ obtained by KKKT with the χ^2 method are significantly (by 2-10 times) lower than their own estimates with the 1σ method.

There is no description of the χ^2- profile analysis in the text of the KKKT article. It is also not clear how many free parameters were used, what is the precise n.d.f, and whether the result depends on the lower and upper limits of the analysis region. The number of the events in the peak ("peak area") seem to have been taken directly from the minimization program together with the error on this number. In principle, the correct limits with acceptable precision can be reproduced by using these data. The results are presented in Table 2. The data of λ are obtained assuming the normal shape for the χ^2- profile with the central value and variation taken from the third column of Table 7 of the KKKT article (these values are reproduced in N_{peak} column of Table 1). Our limits have been calculated by using the Bayesian approach (see i.e. [4], the prior knowledge in our case is the restriction of the positively defined effect). The limits on the life-time recalculated by using these values are presented in the next column. Only the ME case for 68% c.l. is shown, analysis for 90% c.l. and the AE cases can be performed in the similar way with the same conclusions.

As it is seen from our estimations the best limit that can be obtained ($\tau = 3.9 \times 10^{25}$ yr at 68% c.l. for ANG4 setup) is very close to that already existed for the HPGe ($\tau = 3.7 \times 10^{25}$ yr at 68% c.l. [5]). The most surprising fact is that it practically coincides with the estimate of the sensitivity obtained by KKKT themselves with the 1σ method. As a result, the life-time limits in the KKKT article from the χ^2 analysis are by a factor of 2 till 10 stronger than the estimated sensitivity, and by a factor 3 till 5 higher than the values that can be obtained from the values presented as the "Peak area".[3]

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[3] The same is true for the limits on the charge- nonconservation parameter $\epsilon^2_{e \to \nu\gamma}$, derived from the upper

	1σ method			χ^2	
Detector	λ events	τ years	N_{peak} ("peak area")	λ events	τ years
ANG1	49	5.8×10^{24}	-38.187 ± 51.077	13.216	2.146×10^{25}
ANG2	61	3.3×10^{25}	89.444 ± 63.058	38.354	5.285×10^{25}
ANG3	64	2.2×10^{25}	-38.301 ± 67.374	13.216	10.76×10^{25}
ANG4	46	2.0×10^{25}	-76.249 ± 47.401	4.789	19.33×10^{25}
ANG5	73	3.0×10^{25}	-33.273 ± 75.947	14.343	15.69×10^{25}

Table 1: Data from the KKKT article (68% c.l.; ME case only).λ is an upper limit on the number of events in the peak from the hypothetical electron decay, τ is the corresponding life-time limit.

	MC	χ^2	
Detector	λ(68% c.l.) events	λ(68% c.l.) events	τ (68% c.l.) years
ANG1	46	36	7.8×10^{24}
ANG2	58	119	1.7×10^{25}
ANG3	62	51	2.8×10^{25}
ANG4	44	24	3.9×10^{25}
ANG5	71	61	3.7×10^{25}

Table 2: Reestimation of the life-time limits (68% c.l.; ME case only).

In the proper analysis one should take the signal from all electrons (the AE case) into account, not only from the outer shells, that can obviously change the sensitivity. The limit for the AE case is inferior to the existing for the HPGe, so there is no need in a more detailed analysis.

Some words should be written on the model used to describe the underlying background. The quality of the fit for 4 sets is bad ($\chi^2 \simeq 390/280$, this value of χ^2 from the formal statistical point of view, rejects the model with a very high probability) and only for the ANG2 set it has an acceptable quality ($\chi^2 \simeq 280/280$). If the data are obtained under the same conditions (which seems to apply at least to 4 detectors of Setup 1), then the model should give a statistically compatible description for all sets. The quantitative comparison of the data sets can be performed using Fischer's F-distribution $\frac{\chi_2^2}{\chi_1^2} = F(p, \nu, \nu)$ as a significance test, where ν is a number of the degrees of freedom and p is a confidence level (see i.e.[6]). Solving equation $F(p, 280, 280) = 390/280$ with respect to p, one obtains the statistical probability of the data set with lower χ^2: $p = 0.003$. This definitely points on the systematics problem with the data set ANG2[4].

The linearity of the background has not been justified in the KKKT article, moreover, all the compatible sets contain the statistically evident hole in the background just in the place where the effect is searched for.

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limits on the electron life-time. Using the best established limit on the electron life-time ($\tau_{e \to \nu\gamma} > 4.6 \times 10^{26}$ y, 68% c.l. [2]) and the upper limit on the photon mass ($m_\gamma < 7 \times 10^{-19}$ eV [4]), from formula $\epsilon_{e \to \nu\gamma}^2 = \left(\frac{m_\gamma}{m_e}\right)^2 \frac{5.6 \times 10^{-25}}{\tau_{e \to \nu\gamma}[y]}$ [3] one can obtain the best restriction on the parameter $\epsilon_{e \to \nu\gamma}^2 < 0.23 \times 10^{-98}$.

[4]or, if we assume that the linear model of the underlying background is valid, the situation is inverted, and in this case the data for all detectors except ANG2 should have systematic problems. The question what the real situation is should be addressed to the KKKT authors.

Conclusions

Our analysis of the data presented by KKKT in [1] has shown that the upper limit on the electron decay is overestimated by at least a factor of 5. The statistical analysis of the KKKT contains evident errors. Moreover, one of the presented data sets (containing an "indication of a signal" on 1.4 σ) is statistically inconsistent with 4 others, pointing out on possible systematic problems with the experimental data. The model used in [1] to fit the underlying background has the same problems of statistical incompatibility.

The best limit that can be obtained by using the KKKT data is comparable to the one established previously for the HPGe. The KKKT restriction for charge nonconservation $\epsilon^2_{e \to \nu \gamma} < 0.86 \times 10^{-98}$ is not valid either, since it is based on the overestimated electron life-time. Instead of the above the restriction $\epsilon^2_{e \to \nu \gamma} < 2.3 \times 10^{-99}$ at 90% c.l. can be calculated from the modern best limit on the electron life-time $\tau_{e \to \nu \gamma} > 4.6 \times 10^{26}$ yr (90% c.l.) established by the Borexino collaboration.

References

[1] H.V. Klapdor-Kleingrothaus, I.V. Krivosheina, I.V. Titkova. Phys.Lett. **B644** (2007) 109.

[2] H.O. Back, et al., Phys. Lett. **B525** (2002) 29.

[3] P. Belli, R. Bernabei, C.J. Dai, et al., Phys. Rev. **D61** (2000) 117301.

[4] W.M.Yao at al (Particle Data Group), J.Phys. **G33** (2006) 1.

[5] Y. Aharonov, et al., Phys. Rev. **D52** (1995) 3785; Y. Aharonov, et al., Phys. Lett. **B353** (1995) 168.

[6] J.Wolberg, "Data Analysis Using the Method of Least Squares", Springer-Verlag, Berlin Heidelberg, 2006.